Results and Discussion

To analyze the problem just formulated, we have developed a finite element code that employs four-noded Lagrangian elements and a consistent mass matrix. All nonzero elements of the stiffness matrix except those corresponding to shearing are evaluated by using the 2×2 Gaussian quadrature rule, and those corresponding to shearing are evaluated by using the one-point quadrature rule and a shear correction factor of 5/6. The coupled linear ordinary differential equations (9) are integrated numerically by using the unconditionally stable Newmark method (e.g., see Hughes[5]) with $\delta = 0.25$ and $\gamma = 0.5$. The validation of the code is discussed in Ghosh.[6]

The results presented are based on the following values of material parameters. For aluminum, Young's modulus $E = 65$ GPa, Poisson's ratio $\nu = 0.3$, mass density $\rho = 2700$ kg/m^3, and for the G1195 PZT, $E_{11} = E_{22} = E_{33} = 63$ GPa, $\nu_{12} = \nu_{23} = \nu_{13} = 0.3$, $G_{12} = G_{23} = G_{13} = 24.2$ GPa, $\rho = 7600$ kg/m^3, $d_{31} = 16.6$ pm/V, and $\xi_{11} = \xi_{22} = \xi_{33} = 15.2$ nF/m.

For a uniformly distributed load of 10 N/m^2 suddenly applied to the upper surface of the plate at time $t = 0$ and then kept fixed, Fig. 2 shows the time history of the sensor output. The amplitude of the output steadily decreases because of the Rayleigh damping considered; some damping is also introduced by the numerical algorithm. Results plotted in Fig. 3 elucidate that actuators are effective in controlling the vertical deflection of the centroid of the plate when a displacement type controller with a gain of 5×10^{24} V/C-mm is employed. A challenging task is to design multi-input multi-output controller to simultaneously annul the deflections of several points in the plate.

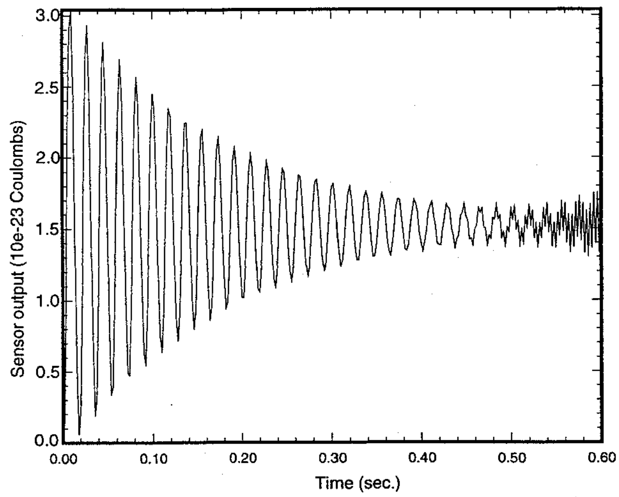

Fig. 2 Time history of the sensor output for the "smart" plate depicted in Fig. 1.

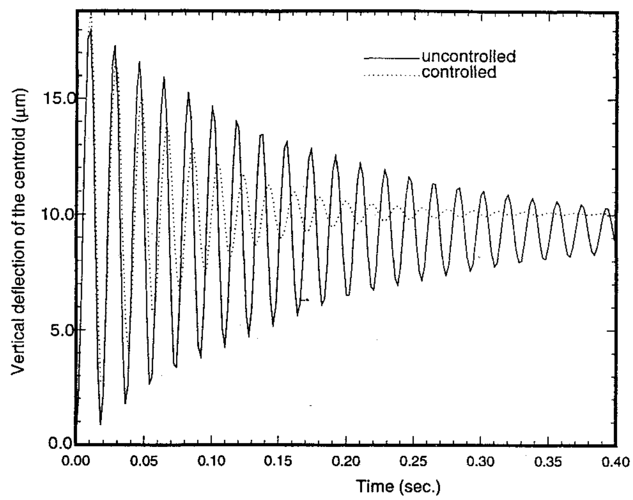

Fig. 3 Time histories of the vertical deflection of the centroid of the plate both with and without activating the actuators.

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References

[1]Baz, A., and Poh, S., "Performance of an Active Control System with Piezoelectric Actuators," *Journal of Sound and Vibration*, Vol. 126, No. 2, 1988, pp. 327–343.

[2]Tzou, H. S., and Tseng, C. I., "Distributed Modal Identification and Vibration Control of Continua: Piezoelectric Finite Element Formulation and Analysis," *Journal of Dynamic Systems, Measurement and Control*, Vol. 113, No. 3, 1991, pp. 501–505.

[3]Crawley, E. F., and de Luis, J., "Use of Piezoelectric Actuators as Elements of Intelligent Structures," *AIAA Journal*, Vol. 25, No. 10, 1987, pp. 1373–1385.

[4]Halpin, J. C., *Primer on Composite Materials: Analysis*, Technomic, Westport, CT, 1984.

[5]Hughes, T. J. R., *The Finite Element Method*, Prentice–Hall, Englewood Cliffs, NJ, 1987, Chaps. 5 and 9.

[6]Ghosh, K., "Shape Control of Plates Using Piezoceramic Elements," M.S. Thesis, Dept. of Mechanical and Aerospace Engineering and Engineering Mechanics, Univ. of Missouri, Rolla, Aug. 1994.

Effective Transverse Young's Modulus of Composites with Viscoelastic Interphase

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Introduction

A NUMBER of studies[1–3] show that the interphase plays an essential role in the performance of fiber-reinforced composites. Papanicolaou et al.[1] reported that the interphase material is viscoelastic and evaluated its mechanical properties. Hashin[2] showed that an imperfect interface affects transverse mechanical properties of composites significantly but not axial Young's modulus. Gosz et al.[3] studied the effect of a viscoelastic interphase on the transverse properties of hexagonal array composites using two Maxwell elements and an elastic matrix.

Previous studies[2,3] carry the assumption of infinitesimally thin interfacial zones. However, the thickness of the interphase may have important consequences in material property characterization and stress distributions. A physical region may exist where material properties are altered due to chemical reaction of the adhesion process between fiber and matrix.

In the present study, time-dependent effects of interfacial layers and matrices on the mechanical properties of laminated composites with various fiber-matrix volume fractions are investigated. Both elastic and viscoelastic constitutive relations are considered for the

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matrix. Two interfacial materials are considered and a small interfacial Young's modulus indicates a weak bond. Finite element methods[4,5] are used to analyze the influence of interphase mechanical properties from a microscopic prospective on overall laminated composite properties.

Analysis

In the present study, the constitutive relations for the fiber are elastic, elastic or viscoelastic for the matrix and viscoelastic for the interphase. For isothermal conditions, rheological stress-strain relationships for viscoelastic transversely isotropic interphase and matrix can be written in the following hereditary integral form:

$$\epsilon_i(\boldsymbol{x}, t) = \int_{-\infty}^{t} S_{ij}(\boldsymbol{x}, t - \tau) \cdot \frac{\partial}{\partial \tau}\sigma_j(\boldsymbol{x}, \tau)\, \mathrm{d}\tau \qquad (1)$$

In Eq. (1), ϵ_j are time-dependent strains and σ_i are stresses. The compliance matrix S_{ij} is related to viscoelastic Young's moduli and to Poisson's ratios.

For composites with a viscoelastic interfacial layer and matrix, volume-average stresses and strains are expressed by

$$\hat{\sigma}_i(\boldsymbol{x}, t) = \frac{1}{V}\int_V \sigma_i(\boldsymbol{x}, t)\, \mathrm{d}V \qquad (3)$$

and

$$\hat{\epsilon}_j(\boldsymbol{x}, t) = \frac{1}{V}\int_V \epsilon_j(\boldsymbol{x}, t)\, \mathrm{d}V \qquad (4)$$

where the superscript caret denotes average properties and V is the composite volume. The effective viscoelastic compliances $\bar{S}_{ij}$ of heterogeneous media are defined by

$$\hat{\epsilon}_i(\boldsymbol{x}, t) = \int_{-\infty}^{t} \bar{S}_{ij}(\boldsymbol{x}, t - \tau) \cdot \frac{\partial}{\partial \tau}\hat{\sigma}_j(\boldsymbol{x}, t)\, \mathrm{d}\tau \qquad (5)$$

where the overbar denotes effective properties. Since the planes x = const are planes of symmetry, the composite material properties are transversely isotropic in the principal material directions.

Consider a fiber-matrix-interphase unit cell (Fig. 1) that, based on an assumed infinite axial dimension, is under generalized plane strain conditions. The generalized plane strain formulation was defined by Lekhnitskii[6] for the first time. Under these conditions, the strain in the x direction is constant, and strain and stress fields in the composite are independent of the x coordinate.

The displacement field for the present analysis is

$$\begin{aligned} u(x, y, z, t) &= x \cdot \epsilon_x(t) + U(y, z, t) \\ v(y, z, t) &= V(y, z, t) \\ w(y, z, t) &= W(y, z, t) \end{aligned} \qquad (6)$$

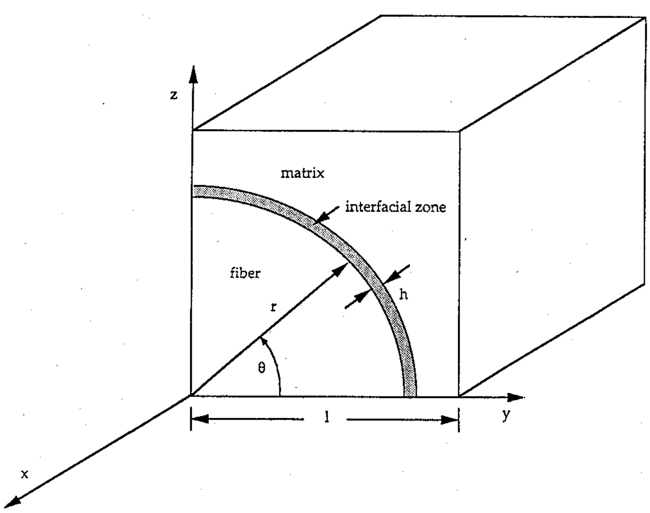

Fig. 1 Schematic of the representative unit cell of composite.

where u is the x displacement, v the y displacement, w the z displacement, and ϵ_x the uniform axial strain.

The displacement field is approximated by the isoparametric interpolation function, and then differentiating these with respect to x_i results in the strain-displacement relationship. Then, by using variational principles and strain-displacement relationships, the finite element equilibrium equations can be obtained. The general finite element formulation and solution procedures for the generalized plane strain viscoelastic boundary value problem have been described by Lin and Yi[4] and Yi.[5] These formulations will be used in the present study.

For the transverse loading, it is assumed that the composite is subjected to a uniform stress σ_y^∞ applied in the y direction at some large distance. Then, from the equilibrium, the stress σ_y^∞ becomes

$$\int_0^l \sigma_y(l, z)\, \mathrm{d}z = \sigma_y^\infty \cdot l \qquad (7)$$

where l is the length of the unit cell.

Numerical Results and Discussion

Several studies have been undertaken to analyze the aforementioned unit cell. Effects of the viscoelastic interfacial layer, matrices, and fiber-matrix volume fractions on the average transverse Young's modulus are investigated.

Two interphase materials are considered. Young's moduli of the interphase materials (IM-1 and IM-2) have the same time variation as illustrated in Fig. 2. The elastic constants of IM-1 are obtained by averaging the fiber and matrix values. A small value of the interfacial material (IM2) properties represents a very weak interface (10 times less stiff than those of matrices). Properties of fiber, matrix, and interfacial materials are given in Table 1.

In a first study, the unit cell is subjected to transverse uniform resultant forces of 1 MPa/m. The interfacial layer is viscoelastic, and the fiber and matrix are elastic. All materials are isotropic. The volume fraction ratio of fiber is 0.5, and thickness of the interphase is $0.05l$. A total 253 four-node isoparametric elements with three degrees of freedom per node are used. Using material IM-1 for the interphase, time-dependent effective transverse Young's moduli are evaluated and shown in Fig. 3. The effective transverse Young's

Table 1 Properties of unit cell components

Component	Young's modulus, Pa	Poisson's ratio
Fiber	6.895×10^{10}	0.2
Matrix	2.758×10^{9}	0.35
Interphase		
IM-1	3.5854×10^{10}	0.25
IM-2	3.5854×10^{8}	0.25

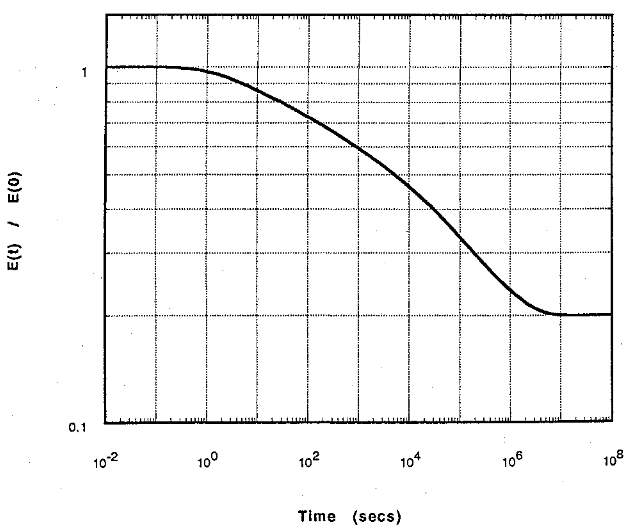

Fig. 2 Normalized time-dependent Young's modulus.

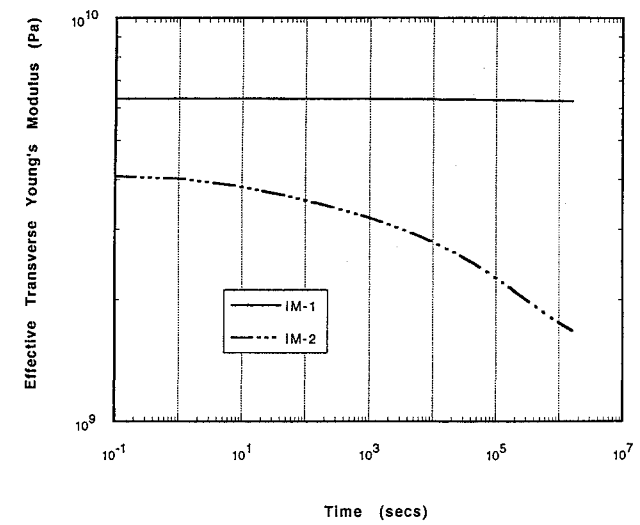

Fig. 3 Effective transverse Young's modulus of composite with elastic matrix.

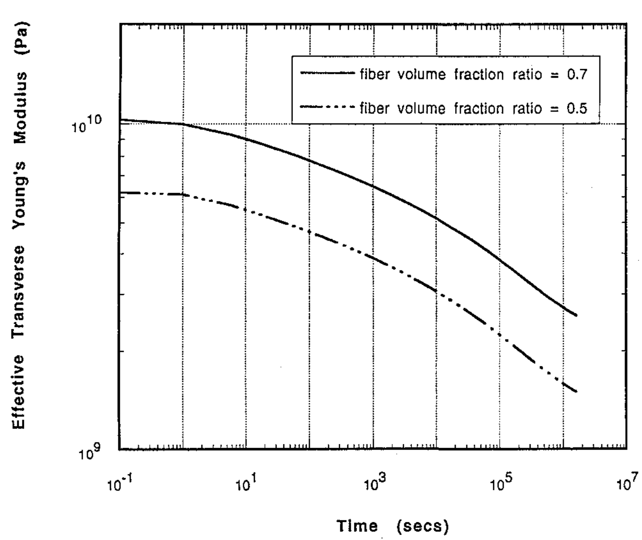

Fig. 4 Effective transverse Young's modulus of composite with viscoelastic matrix.

modulus drops by only 2% with an 80% decrease in the interfacial modulus. However, with material IM-2 representing the interphase, the value of the transverse effective Young's modulus decreases by 58.8% during 1.6×10^6 s time period. The effective transverse Young's modulus is plotted in Fig. 3. In this case, the strains in the fiber decrease with time whereas those in the matrix increase. As time grows, the modulus of the interphase degrades to a much smaller value than that of the fiber or matrix. The interface becomes very weak, and this could be interpreted as approaching a state of debonding.

In a second study, the constitutive relations for the interfacial layer and the matrix are viscoelastic, and the fiber is elastic. The volume ratio of the fiber is 0.7, and the interphase thickness is $0.05l$. The number of elements and degrees of freedom are the same as those in the first study. IM-1 is used for the interfacial material. Time-dependent responses in the fiber, interphase, and matrix are calculated. Transverse strain in the matrix increases 56% during a 1.5×10^6 s period whereas the Young's modulus of the matrix decreases by 78%. Another study comprises a fiber-reinforced composite with volume fraction $\nu_f = 0.5$. Fiber, interphase, and matrix properties are given in the second example. Effective Young's moduli in the transverse direction are computed and are depicted in Fig. 4. The effective transverse Young's modulus is significantly affected by the fiber volume fraction and viscoelastic behavior of the matrix.

References

[1]Papanicolaou, G. C., Paipetis, S. A., and Theocaris, P. S., "The Concept of the Boundary Interphase in Composite Mechanics," *Colloid and Polymer Science*, Vol. 256, July 1978, pp. 333–348.

[2]Hashin, Z., "Thermoelastic Properties of Fiber Composites with Imperfect Interface," *Mechanics of Materials*, Vol. 8, 1990, pp. 333–348.

[3]Gosz, M., Moran, B., and Achenbach, J. D., "Effect of a Viscoelastic Interface on the Transverse Behavior of Fiber-reinforced Composites," *International Journal of Solids and Structures*, Vol. 27, No. 14, 1991, pp. 1757–1771.

[4]Lin, K. Y., and Yi, S., "Analysis of Interlaminar Stresses in Viscoelastic Composites," *International Journal of Solids and Structures*, Vol. 27, No. 7, 1991, pp. 929–945.

[5]Yi, S., "Thermoviscoelastic Analysis of Delamination Onset and Free Edge Response in Epoxy Matrix Composite Laminates," *AIAA Journal*, Vol. 31, No. 12, 1993, pp. 2302–2328.

[6]Lekhnitskii, S. G., *Theory of Elasticity of an Anisotropic Body*, Holden-Day, San Francisco, CA, 1963.

Submodeling Approach to Adaptive Mesh Refinement

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Introduction

ERRORS exist in finite element results in regions where the underlying piecewise polynomials cannot replicate the exact solution. Zienkiewicz and Zhu[1] have shown that these discretization errors can be estimated by comparing the discontinuous finite element stress fields to smoothed stress fields developed from the finite element results. The errors in individual elements are quantified as the strain energy contained in the differences between the two stress fields.

The finite element solution is improved by refining the mesh in regions of high error and solving the problem again. This adaptive refinement process is repeated until an acceptable level of accuracy is achieved. In the Zienkiewicz and Zhu approach, this iterative process is terminated when the global error, which is computed as the sum of the elemental errors, reaches a prespecified level.

This paper presents two improvements to the adaptive refinement scheme just described: 1) instead of reanalyzing the whole region, only regions of interest such as stress concentrations are reanalyzed and 2) instead of terminating the adaptive refinement using a criterion based on the global error, a local error criterion is used. The first modification has the advantage of focusing the computational efforts where they belong. The second modification has the advantage of insuring that the solution is accurate in critical regions.

The first improvement is accomplished by identifying an internal boundary that passes through elements with a low level of error. The subproblem defined by this process is refined, the displacements from the previous analysis are imposed on the internal boundary, and the new model is solved. The second improvement terminates the local adaptive refinement process when the elemental errors in the subproblem reach a prespecified level. These changes improve the efficiency of the analysis and ensure that the accuracy of the results in the critical region is acceptable. All of the computations, including the automated mesh refinements, are performed within a commercial finite element code.

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